Cambridge Assessment International Education

# Cambridge International AS & A Level

**FURTHER MATHEMATICS** **9231/04**

Paper 4 Further Probability & Statistics **For examination from 2020**

MARK SCHEME

Maximum Mark: 50

**Specimen**

This document has **8** pages. Blank pages are indicated.

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## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

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GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

**Mark Scheme Notes**

Marks are of the following three types.

| | |
|---|---|
| M | Method mark, given for a valid method applied to the problem. Method marks can still be given even if there are numerical errors, algebraic slips or errors in units. However the method must be applied to the specific problem, e.g. by substituting the relevant quantities into a formula. Correct use of a formula without the formula being quoted earns the M mark and in some cases an M mark can be implied from a correct answer. |
| A | Accuracy mark, given for an accurate answer or accurate intermediate step following a correct method. Accuracy marks cannot be given unless the relevant method mark has also been given. |
| B | Mark for a correct statement or step. |
| DM or DB | M marks and B marks are generally independent of each other. The notation DM or DB means a particular M or B mark is dependent on an earlier M or B mark (indicated by *). When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given. |

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT below).
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures (sf) or would be correct to 3 sf if rounded (1 decimal place (dp) in the case of an angle in degrees). As stated above, an A or B mark is not given if a correct numerical answer is obtained from incorrect working.
- Common alternative solutions are shown in the Answer column as: '**EITHER** Solution 1 **OR** Solution 2 **OR** Solution 3 …'. Round brackets appear in the Partial Marks column around the marks for each alternative solution.
- The total number of marks available for each question is shown at the bottom of the Marks column in bold type.
- Square brackets [ ] around text show extra information not needed for the mark to be awarded.

The following abbreviations may be used in a mark scheme.

| | |
|---|---|
| AG | Answer given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid). |
| CAO | Correct answer only (emphasising that no 'follow through' from an error is allowed). |
| CWO | Correct working only |
| FT | Follow through after error (see Mark Scheme Notes for further details). |
| ISW | Ignore subsequent working |
| OE | Or equivalent form |
| SC | Special case |
| SOI | Seen or implied |

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| Question | Answer | Marks | Partial Marks | Guidance |
|---|---|---|---|---|
| 1(a) | When the population cannot be assumed to be normally distributed | **1** | **B1** | |
| 1(b) | $H_0$: population median is 6.00, $H_1$: population median is greater than 6.00 | 1 | **B1** | Both hypotheses stated |
| | Calculate deviations and resulting signed ranks | 1 | **M1** | |
| | Devs: –0.38, –0.27, 0.55, 0.81, 0.10, –0.25, –0.13, 0.47, –0.14, 0.26, 0.99, –0.09<br>Rank: –8, –7, 10, 11, 2, –5, –3, 9, –4, 6, 12, –1 | 1 | **A1** | |
| | Test statistic $T = 8 + 7 + 5 + 3 + 4 + 1 = 28$ | 1 | **A1** | |
| | Compare with correct critical value 17 | 1 | **M1** | |
| | Conclusion: accept $H_0$; insufficient evidence that the median is greater than 6.00 | 1 | **A1FT** | Conclusion to be stated in context, not just 'not significant'; follow through their value of $T$ |
| | | **6** | | |

| Question | Answer | Marks | Partial Marks | Guidance |
|---|---|---|---|---|
| 2 | Use Geo(0.6) to calculate expected values | 1 | **M1** | Need at least 3 values correct |
| | 120, 48, 19.2, 7.68, 3.072, 1.2288, 0.8192 | 1 | **A1** | All 7 correct, stated or implied |
| | Last 3 cells combined | 1 | **M1** | Observed frequency = 6, expected frequency = 5.12 |
| | $\frac{(126-120)^2}{120} + \frac{(43-48)^2}{48} + \frac{(22-19.2)^2}{19.2} + \frac{(3-7.68)^2}{7.68} + \frac{(6-5.12)^2}{5.12}$ | 1 | **M1** | 0.3 + 0.5208 + 0.4083 + 2.8519 + 0.15125 |
| | $\chi^2 = 4.23$ | 1 | **A1** | Correct value to (at least) 3 sf |
| | Use appropriate tabular value for their test | 1 | **M1** | |
| | $4.23 < 9.49$ so the distribution Geo(0.6) is a satisfactory fit | 1 | **A1** | Correct conclusion from correct values; conclusion must refer to the distribution explicitly or via a statement of the null hypothesis for the test |
| | | **7** | | |

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| Question | Answer | Marks | Partial Marks | Guidance |
|---|---|---|---|---|
| 3 | Assume that population differences are normally distributed | 1 | **B1** | |
| | $H_0 : \mu_B - \mu_A = 0$ and $H_1 : \mu_B - \mu_A > 0$ | 1 | **B1** | Both hypotheses required, in terms of population means or alternatively in terms of population mean difference |
| | Differences, $d$: 8 3 –4 2 –11 3 17 10 6 0 | 1 | **M1** | |
| | $\overline{d} = 3.4$ and $s^2 = \frac{1}{9}\left(648 - \frac{34^2}{10}\right) = 59.16$ | 1 | **M1** | |
| | Calculate test statistic $t = \frac{\overline{d}}{s/\sqrt{10}}$ | 1 | **M1** | |
| | $t = 1.398$ | 1 | **A1** | Correct value to at least 3 sf |
| | Use tabular value $t_{9,\,0.9} = 1.383$ in a comparison | 1 | **B1** | |
| | Reject $H_0$ and conclude that hours of absence have decreased | 1 | **B1FT** | Follow through their calculated $t = 1.398$ |
| | | **8** | | |

| Question | Answer | Marks | Partial Marks | Guidance |
|---|---|---|---|---|
| 4 | Sample variances are $\frac{4341.6}{59}$ and $\frac{3732.0}{49}$ | 1 | **M1** | Either attempted |
| | $s_x^2 = 73.59$ and $s_y^2 = 76.16$ | 1 | **A1** | Both correct to at least 2 dp |
| | Variance for difference of means: $s^2 = \frac{s_x^2}{60} + \frac{s_y^2}{50}$ | 1 | **M1** | |
| | $s^2 = 2.750$ or $s = 1.658$ | 1 | **A1** | Correct value to at least 4 sf |
| | Evaluate $(\overline{x} - \overline{y}) \pm zs$ | 1 | **M1** | Any $z$-value, and using their value of $s$ |
| | Use of correct $z = 1.96$ | 1 | **A1** | |
| | End-points of confidence interval are 1.55 and 8.05 | 1 | **A1** | Accept e.g. (1.55, 8.05) or $1.55 < \mu_A - \mu_B < 8.05$ |
| | | **7** | | |

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| Question | Answer | Marks | Partial Marks | Guidance |
|---|---|---|---|---|
| 5(a) | $P(X > 1) = 1 - \frac{1}{2} \times 1 \times \frac{6}{5} = \frac{2}{5}$<br>**OR**<br>via $\int_1^\infty \frac{6}{5}x^{-4}\mathrm{d}x$ | **1** | **B1** | |
| 5(b) | Median satisfies $\frac{1}{2} \times m \times \frac{6}{5}m = \frac{1}{2}$<br>**OR** equivalent use of integration | 1 | **M1** | |
| | $m = \sqrt{\frac{5}{6}}$ | 1 | **A1** | Correct value; accept 0.913 |
| | | **2** | | |
| 5(c) | $E(X^2) = \int_0^1 \frac{6}{5}x^3\mathrm{d}x + \int_1^\infty \frac{6}{5}x^{-2}\mathrm{d}x$ | 1 | **M1** | Attempt evaluation of the sum of these two integrals |
| | $E(X^2) = 1.5$ | 1 | **A1** | Correct value seen or implied |
| | $\mathrm{Var}(X) = 1.5 - 1^2 = 0.5$ | 1 | **A1FT** | Follow through their 1.5 providing variance is positive |
| | | **3** | | |
| 5(d) | $E(\sqrt{X}) = \int_0^1 \frac{6}{5}x^{\frac{3}{2}}\mathrm{d}x + \int_1^\infty \frac{6}{5}x^{-\frac{7}{2}}\mathrm{d}x$ | 1 | **M1** | Attempt evaluation of the sum of these two integrals |
| | $\frac{24}{25}$ or 0.96 | 1 | **A1** | |
| | | **2** | | |

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| Question | Answer | Marks | Partial Marks | Guidance |
|---|---|---|---|---|
| 6(a) | Probabilities $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$ for 0, 1, 2, 3 reds | 1 | **B1** | |
| | $G_X(t) = \frac{1}{8} + \frac{3}{8}t + \frac{3}{8}t^2 + \frac{1}{8}t^3$ | 1 | **B1FT** | Follow through their probabilities so long as $\Sigma p = 1$ |
| | | **2** | | |
| 6(b) | Correct value $\frac{9}{20}$ or 0.45 for $P(Y = 1)$ or $P(Y = 2)$ | 1 | **B1** | |
| | Probabilities $\frac{1}{20}, \frac{9}{20}, \frac{9}{20}, \frac{1}{20}$ for 0, 1, 2, 3 reds | 1 | **B1** | |
| | $G_Y(t) = \frac{1}{20} + \frac{9}{20}t + \frac{9}{20}t^2 + \frac{1}{20}t^3$ | 1 | **B1FT** | Follow through their probabilities so long as $\Sigma p = 1$ |
| | | **3** | | |
| 6(c) | $G_Z(t) = G_X(t) \times G_Y(t)$ | 1 | **B1** | Stated or implied |
| | Complete expansion of the product of their cubics | 1 | **M1** | |
| | $G_Z(t) = \frac{1}{160}(1 + 12t + 39t^2 + 56t^3 + 39t^4 + 12t^5 + t^6)$ | 1 | **A1** | Or equivalent, e.g. with explicit fractional coefficients |
| | | **3** | | |
| 6(d) | Attempt to differentiate $G_Z(t)$ and evaluate $G'_Z(1)$ | 1 | **M1** | |
| | $E(Z) = 3$ | 1 | **A1** | Correctly obtained from $G'_Z(1)$ |
| | Attempt to find second derivative $G''_Z(t)$ | 1 | **M1** | |
| | Use of $G''_Z(1) + G'_Z(1) - \left(G'_Z(1)\right)^2$ | 1 | **M1** | |
| | $\text{Var}(Z) = 1.2$ | 1 | **A1** | Correct value correctly obtained |
| | | **5** | | |

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